Math 60 12.1 Distance and Midpoint Formulas - Day 1

Objectives

1) Use the distance formula to find
   - the distance between 2 points
   - the length of a line segment connecting two points

   } the same Day 1

2) Use the midpoint formula to find
   - The coordinates of an ordered pair which is exactly halfway between 2 points.
   - The coordinates of the midpoint of a line segment connecting two points

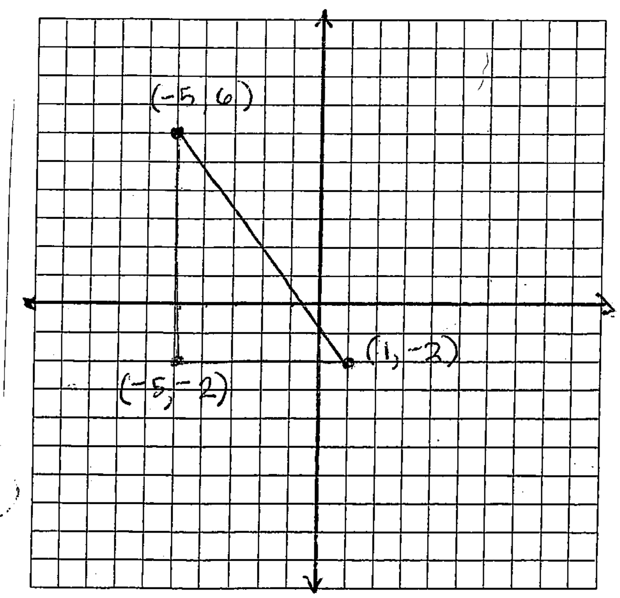
   } the same Day 2

① GOAL: Find the distance between $(-5, 6)$ and $(1, -2)$.

EXPLORE (The long way, to understand where we get the distance formula.)

Graph the two points.
Connect them with a line.
Draw a right triangle.

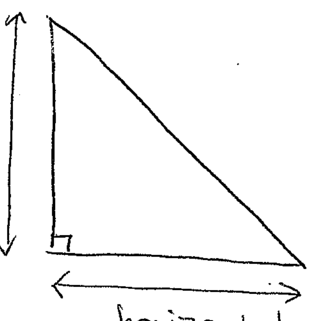
Our goal is to find the length of the hypotenuse of this right triangle.



vertical length uses y-coordinates

$(-2-6)^2 = 8^2$

$(y_2-y_1)^2 = b^2$



horizontal length uses x-coordinates

$(1-(-5))^2 = (6)^2 = a^2$

$(x_2-x_1)^2 = a^2$

Pythagorean Theorem

$a^2+b^2=c^2$

$6^2+8^2=c^2$

$(1-(-5))^2+(-2-6)^2=c^2$

$\sqrt{(1-(-5))^2+(-2-6)^2}=c$

$\text{distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$c^2 = 36+64$

$c^2 = 100$

$c=\sqrt{100}$

$c = 10.$

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* The distance between (x_1, y_1) and (x_2, y_2), D is given by $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Memorize!

(2) Use the distance formula to find the distance between $(-2,-1)$ and $(4,3)$

Step 1: Write the distance formula

$$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Step 2: Decide which point is (x_1, y_1) and which is (x_2, y_2). and substitute.

$(-2,-1)$ (x_1, y_1) $\quad$ $(4, 3)$ (x_2, y_2)

$$D = \sqrt{(4-(-2))^2 + (3-(-1))^2}$$

step 3: Use the order of operations

$D = \sqrt{(4+2)^2 + (3+1)^2}$ inside ( )

$= \sqrt{6^2 + 4^2}$ exponents

$= \sqrt{36+16}$ add

$= \sqrt{52}$ simplify radical

$= \sqrt{4 \cdot 13}$

$= \boxed{2\sqrt{13}}$

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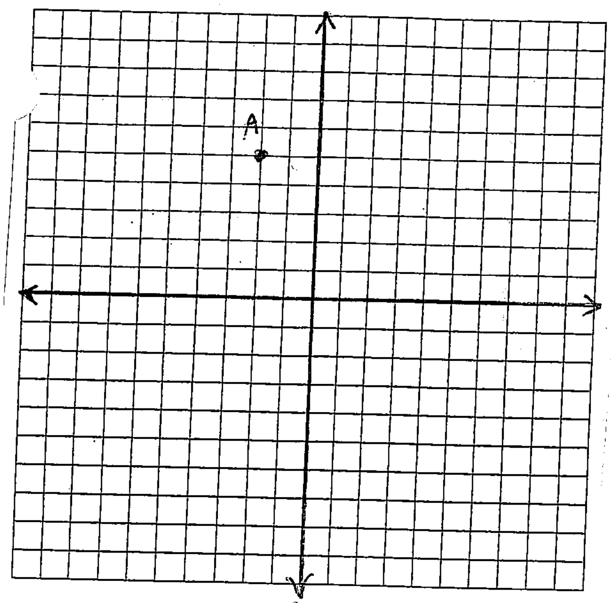
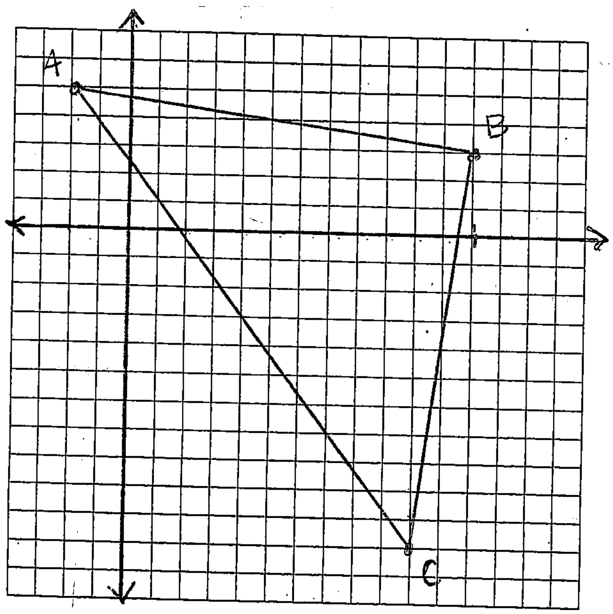
a) Plot points on a graph and draw $\triangle ABC$
b) Find lengths of each side of $\triangle$
c) Verify that $\triangle ABC$ is a right $\triangle$.
d) Find the area of the triangle.

(3) Consider the points A $(-2,5)$
B $(12,3)$
C $(10,-11)$

a) Plot the points on a graph and draw $\triangle ABC$.

Woops! B and C won't fit on my grid if I leave the axes here...





b) Find the length of each side of the triangle $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\overline{AB}$ use $(-2,5)$ and $(12,3)$ $\sqrt{(12+2)^2+(3-5)^2}$

$$= \sqrt{14^2+(-2)^2}$$
$$= \sqrt{196+4}$$
$$= \sqrt{200}$$
$$= \sqrt{100}\cdot\sqrt{2}$$
$$= \boxed{10\sqrt{2}}$$

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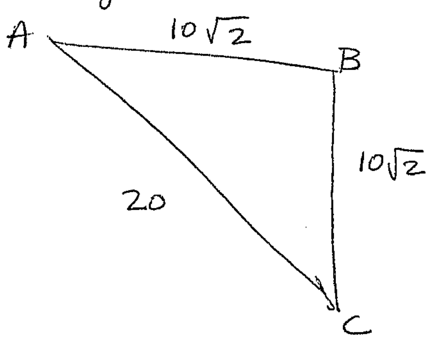
$\overline{BC}$ use $(12,3)$ and $(10,-11)$

$$\sqrt{(12-10)^2+(3+11)^2}$$
$$=\sqrt{2^2+14^2}$$
$$=\sqrt{4+196}$$
$$=\sqrt{200}$$
$$=\sqrt{100}\cdot\sqrt{2}$$
$$=\boxed{10\sqrt{2}}$$

$\overline{AC}$ use $(-2,5)$ and $(10,-11)$

$$\sqrt{(10+2)^2+(-11-5)^2}$$
$$=\sqrt{12^2+(-16)^2}$$
$$=\sqrt{144+256}$$
$$=\sqrt{400}$$
$$=\boxed{20}$$

c) Verify that the triangle is a right Δ.



$10\sqrt{2} \approx 14.1$

If the triangle is a right Δ, the longest side 20 is the hypotenuse $\Rightarrow$ this means angle B is the right angle.

Method 1: Show that the sides satisfy the Pythagorean Theorem.

If ΔABC is a right Δ then $a^2+b^2=c^2$ — but ALSO:

If $a^2+b^2=c^2$ then ΔABC is a right Δ.

$$(10\sqrt{2})^2+(10\sqrt{2})^2 \stackrel{?}{=} (20)^2$$

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$$100 \cdot 2 + 100 \cdot 2 = 400$$

$$200 + 200 = 400. \checkmark \quad \text{Yes.}$$

This is the method the book expects you to use.

Method 2: Show that the segment $\overline{AB}$ is perpendicular to the segment $\overline{BC}$ because the slopes of these lines are opposites and reciprocals.

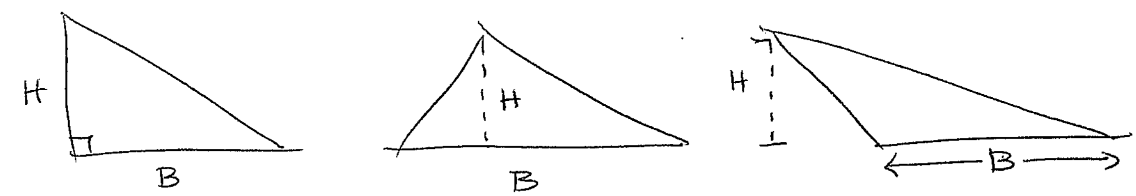
$$\text{slope } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-2-(12)} = \frac{2}{-14} = \frac{-1}{7}$$

$$\text{slope } BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-11)}{12-10} = \frac{14}{2} = 7$$

yes $\frac{-1}{7}$ is the opposite and reciprocal of 7.

d) Find the area of the triangle.

Recall: Formula for area of Δ. $A = \frac{1}{2} B \cdot H$



Base and height must be perpendicular – right angles.

For our triangle $\overline{AB} \perp \overline{BC}$ so these two segments are our height and base

$$\text{Area} = (10\sqrt{2})(10\sqrt{2})$$
$$= 10 \cdot 10 \cdot \sqrt{2} \cdot \sqrt{2}$$
$$= 100 \cdot 2$$
$$= \boxed{200}$$